education equals

Topic: Assignment Problems

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [3, 4: 7 marks]

a) The following table show the number of vehicles which will be effected by road works on road A, B and C depending on the rerouting of traffic via route 1, 2 or 3.

Assign roads A, B and C to the rerouting route in order to minimise the number of vehicles effected.

State which road should use which route and the total number of vehicles expected to be effected.

	Route 1	Route 2	Route 3
Road A	110	200	260
Road B	100	300	210
Road C	190	250	220

b) A local government has projects A, B, C and D which need to be completed. They have quotes from four contractors and need to assign each contractor one project.

Which project should be assigned to each contractor in order to minimise costs to the government and what is the total cost of project A, B, C and D.

Costs for each project by each contractor are shown in the table below in $1 000's.

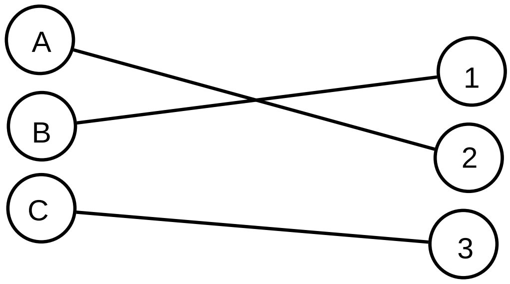
	Contractor 1	Contractor 2	Contractor 3	Contractor 4
Project A	360	210	100	190
Project B	250	220	200	90
Project C	250	200	240	210
Project D	205	250	300	200

Question Two: [1, 3, 8: 12 marks]

Four people needing a kidney transplant and four willing donors are ranked according to the potential for a successful transplant. 0 being not a successful match and 100 being a perfect match. The following table shows a summary of the results.

	Donor 1	Donor 2	Donor 3
Recipient A	26	29	30
Recipient B	58	55	56
Recipient C	75	78	69

a) If a computer randomly allocated recipients to donors and the allocation is shown below, calculate the potential success rate out of 300.



b) Is there a better allocation of recipients to donors in order to maximise the chance of success?

c) If another recipient and another donor are included in the process show how the Hungarian algorithm can be used to identify the maximum chances of success by matching the recipients to the donors.

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A	26	29	30	25
Recipient B	58	55	56	60
Recipient C	75	70	69	70
Recipient D	66	68	68	65

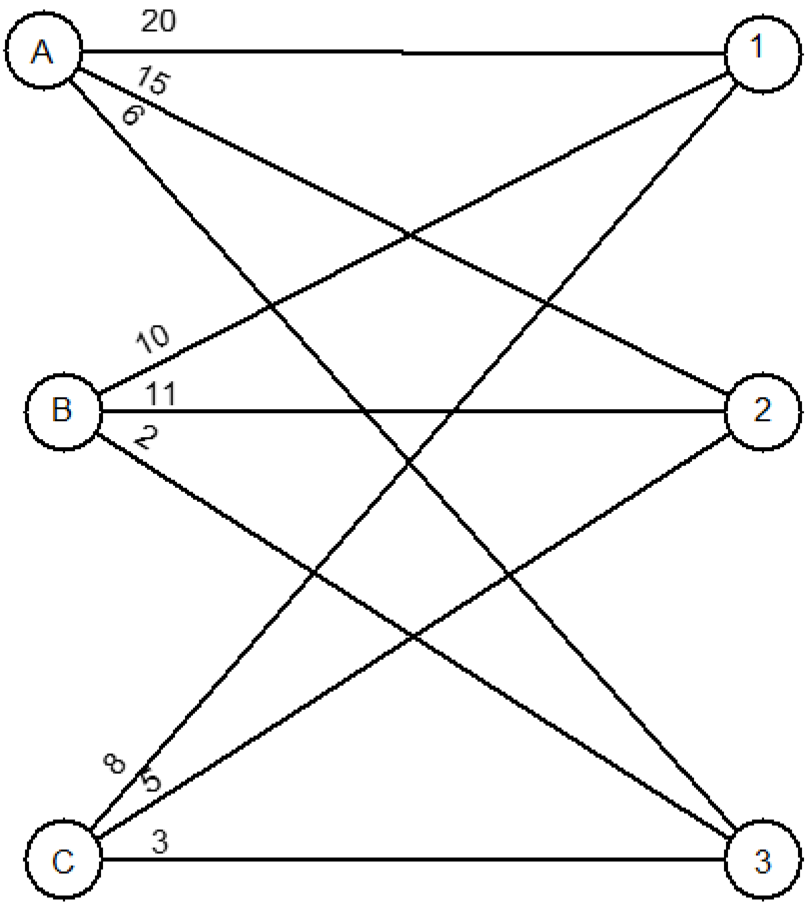
	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A				
Recipient B				
Recipient C				
Recipient D				

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A				
Recipient B				
Recipient C				
Recipient D				

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A				
Recipient B				
Recipient C				
Recipient D				

Question Three: [4, 3: 7 marks]

a) Find the perfect matching of minimum weighting for the following complete bipartite graph.



b) Find the perfect matching of maximum weighting for the complete bipartite graph above.

Question Four: [1, 3, 3: 7 marks]

The following table displays the delivery costs for companies A, B, C and D to complete four deliveries.

	1	2	3	4
A	100	220	350	110
B	90	200	300	100
C	110	190	360	150
D	150	140	310	120

The Hungarian algorithm was being used to find the allocation which would minimise delivery costs.

The first step of the Hungarian algorithm is shown in the partially completed table below.

a) Complete this step by completing the table.

	1	2	3	4
A	0	120	250	10
B	0	110	210	10
C	0	80	250	40
D				

Further along in the process of the Hungarian algorithm the table is as follows:

	1	2	3	4
A	0	100	60	10
B	0	90	20	10
C	0	60	60	40
D	30	0	0	0

b) Complete the final stages of the Hungarian algorithm to determine the minimum allocation.

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

c) Indicate the allocation on the original table and state the total cost of the four deliveries.

Question Five: [1, 4, 4, 3: 12 marks]

Consider the following table:

	1	2	3
A	$x+4$	$x+2$	x
B	$x+3$	$2x$	$x+4$
C	$x+1$	$2x-1$	$2x-1$

a) If x is an integer > 4, which expression represents the highest value?

b) If we want to maximise the allocation on the table above show the first step of the Hungarian algorithm in the table below.

	1	2	3
A			
B			
C			

c) Show that the final table for maximizing this allocation using the Hungarian algorithm is shown in the table below.

	1	2	3
A	0	2	4
B	$x-3$	0	$x-4$
C	$x-2$	0	0

The sum of the final allocation which maximise the result is 33.

d) Calculate the value of x.

education equals

Topic: SOLUTIONS

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [3, 4: 7 marks]

a) The following table show the number of vehicles which will be effected by road works on road A, B and C depending on the rerouting of traffic via route 1, 2 or 3.

Assign roads A, B and C to the rerouting route in order to minimise the number of vehicles effected.

State which road should use which route and the total number of vehicles expected to be effected.

	Route 1	Route 2	Route 3
Road A	110	200	260
Road B	100	300	210
Road C	190	250	220

Road A – Route 2 ✓

Road B – Route 1 ✓ Total vehicles effected 520 ✓

Road C – Route 3

b) A local government has projects A, B, C and D which need to be completed. They have quotes from four contractors and need to assign each contractor one project.

Which project should be assigned to each contractor in order to minimise costs to the government and what is the total cost of project A, B, C and D.

Costs for each project by each contractor are shown in the table below in $1 000's.

	Contractor 1	Contractor 2	Contractor 3	Contractor 4
Project A	360	210	100	190
Project B	250	220	200	90
Project C	250	200	240	210
Project D	205	250	300	200

PA – C3, PB – C4, PC – C2, PD – C1
✓ ✓ ✓

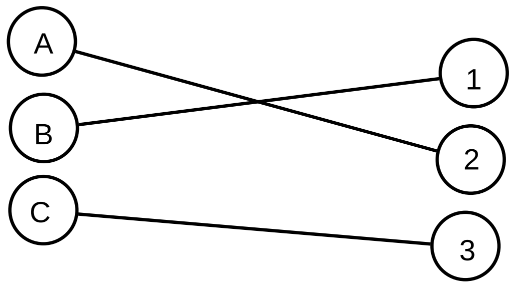
Total cost $595 000
✓

Question Two: [1, 3, 8: 12 marks]

Four people needing a kidney transplant and four willing donors are ranked according to the potential for a successful transplant. 0 being not a successful match and 100 being a perfect match. The following table shows a summary of the results.

	Donor 1	Donor 2	Donor 3
Recipient A	26	29	30
Recipient B	58	55	56
Recipient C	75	78	69

a) If a computer randomly allocated recipients to donors and the allocation is shown below, calculate the potential success rate out of 300.



rate $= \frac{156}{300}$ ✓

b) Is there a better allocation of recipients to donors in order to maximise the chance of success?

Yes ✓ A – 3 → 30

B – 1 → 58 ✓

C – 2 → 78 rate $= \frac{166}{300}$ ✓

c) If another recipient and another donor are included in the process show how the Hungarian algorithm can be used to identify the maximum chances of success by matching the recipients to the donors.

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A	26	29	30	25
Recipient B	58	55	56	60
Recipient C	75	70	69	70
Recipient D	66	68	68	65

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A	49	46	45	50
Recipient B	17	20	19	15
Recipient C	0	5	6	5
Recipient D	9	7	7	10

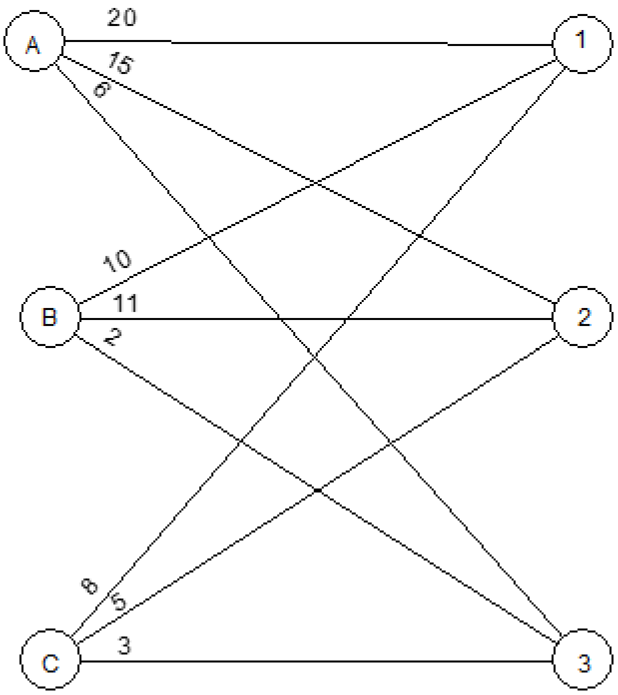
	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A	4	1	0	5
Recipient B	2	5	4	0
Recipient C	0	5	6	5
Recipient D	2	0	0	3

	Donor 1	Donor 2	Donor 3	Donor 4
Recipient A	4	1	0	5
Recipient B	2	5	4	0
Recipient C	0	5	6	5
Recipient D	2	0	0	3

D1 – C
D2 –D
D3 – A
D4 – B

Question Three: [4, 3: 7 marks]

a) Find the perfect matching of minimum weighting for the following complete bipartite graph.



	A	B	C
1	~~20~~ ~~12~~ 8	~~15~~ ~~10~~ 6	~~6~~ ~~4~~ 0
2	~~10~~ 2	~~11~~ 6	~~2~~ 0
3	~~8~~ 0	~~5~~ 0	~~3~~ 1

	A	B	C
1	~~8~~ 6	~~6~~ 4	0
2	~~2~~ 0	~~6~~ 4	0
3	0	0	~~1~~ 3

A – 3 6

B – 1 10 21

C – 2 5

b) Find the perfect matching of maximum weighting for the complete bipartite graph above.

	A	B	C
1	~~20~~ 0	~~15~~ 5	~~6~~ 14
2	~~10~~ 10	~~11~~ 9	~~2~~ 18
3	~~8~~ 12	~~5~~ 15	~~3~~ 17

	A	B	C
1	0	5	~~14~~ 9
2	~~10~~ 1	~~9~~ 0	~~18~~ ~~9~~ 4
3	~~12~~ 0	~~15~~ 3	~~17~~ ~~5~~ 0

A – 1 20

B – 2 11 34

C – 3 3

Question Four: [1, 3, 3: 7 marks]

The following table displays the delivery costs for companies A, B, C and D to complete four deliveries.

	1	2	3	4
A	100	220	350	110
B	90	200	300	100
C	110	190	360	150
D	150	140	310	120

The Hungarian algorithm was being used to find the allocation which would minimise delivery costs.

The first step of the Hungarian algorithm is shown in the partially completed table below.

a) Complete this step by completing the table.

	1	2	3	4
A	0	120	250	10
B	0	110	210	10
C	0	80	250	40
D	30	20	190	0

✓

Further along in the process of the Hungarian algorithm the table is as follows:

	1	2	3	4
A	0	100	60	10
B	0	90	20	10
C	0	60	60	40
D	30	0	0	0

b) Complete the final stages of the Hungarian algorithm to determine the minimum allocation.

	1	2	3	4
A	0	~~90~~ 80	~~50~~ 40	0
B	0	~~80~~ 70	~~10~~ 0	0
C	0	~~50~~ 40	~~50~~ 40	30
D	40	0	0	10

	1	2	3	4
A				0
B			0	
C	0			
D		0		

c) Indicate the allocation on the original table and state the total cost of the four deliveries.

A – 4 110

B – 3 300

C – 1 110

D – 2 140

Total cost $660

Question Five: [1, 4, 4, 3: 12 marks]

Consider the following table:

	1	**2**	**3**
A	$x+4$	$x+2$	x
B	$x+3$	$2x$	$x+4$
C	$x+1$	$2x-1$	$2x-1$

a) If x is an integer > 4, which expression represents the highest value? $2x$ ✓

b) If we want to maximise the allocation on the table above show the first step of the Hungarian algorithm in the table below.

	1	**2**	**3**
A	$2x-x-4$ $=x-4$ ✓	$2x-x-2$ $=x-2$	$2x-x=x$ ✓
B	$2x-x-3$ $=x-3$	0 ✓	$2x-x-4$ $=x-4$
C	$2x-x-1$ $=x-1$	$2x-2x+1=1$	1 ✓

c) Show that the final table for maximizing this allocation using the Hungarian algorithm is shown in the table below.

	1	**2**	**3**
A	0	2	4
B	$x-3$	0	$x-4$
C	$x-2$	0	0

✓ $x-4\ (smallest)$ $\quad x-2 \quad x$

$x-3 \quad 0 \quad x-4$

$x-1 \quad 1\ (smallest)$ ✓ $\quad 1$

$x-4-(x-4)=0 \quad x-2-(x-4)=2 \quad x-(x-4)=4$ ✓

$x-3 \quad 0 \quad x-4$ ✓

$x-1-1=x-2 \quad 1-1=0 \quad 1-1=0$

The sum of the final allocation which maximise the result is 33.

d) Calculate the value of *x*.

$A - 1\ x + 4$

$B - 2\ 2x$ ✓

$C - 3\ 2x - 1$

$x + 4 + 2x + 2x - 1 = 33$ ✓

$5x + 3 = 33$

$5x = 30$

$x = 6$ ✓

	1	2	3
A	10	8	6
B	9	12	10
C	7	11	11